PROOF BY MATHEMATICAL INDUCTION

Most proofs by mathematical induction share several parts that have the same structure.

To prove a statement is true for all integers $n \geq 1$ (*):

1. Basis step: Prove the statement is true when $n = 1$ (*).

2. Inductive step:

   [a] Assume the statement is true for some particular but arbitrary integer $k \geq 1$ (*) (ie. when $n = k$).

   It is helpful to explicitly write down the statement when *n* = *k*, so you know what you're allowed to assume and use.

   [b] Prove the statement is true when $n = k + 1$.

   It is helpful to explicitly write down the statement when *n* = *k* + 1, so you know what you're trying to prove.

The proof in part 2[b] is different for each proof.
A frequent pattern of proving that part is to try to

[i] rewrite a complex expression from step 2[b] so that the similar expression from step 2[a] appears
[ii] use the statement from 2[a] to make a statement using a slightly simpler expression
[iii] rewrite the slightly simpler expression so that the simpler expression from step 2[b] appears

(*) To prove a statement is true for all integers n ≥ some other number, replace these occurrences of 1 with that other number.

For each example below,

1. What are you supposed to prove is true in the basis step ?

2. [a] What are you supposed to assume is true in the inductive step ?

   [b] What are you supposed to prove is true in the inductive step ?

Example 1 $\dfrac{1}{1 \times 4} + \dfrac{1}{4 \times 7} + \dfrac{1}{7 \times 10} + \cdots + \dfrac{1}{(3n-2) \times (3n+1)} = \dfrac{n}{3n+1}$

Example 2 $\displaystyle\sum_{i=1}^{n} i \cdot 2^i = (n-1)2^{n+1} + 2$

Example 3 $\displaystyle\sum_{i=1}^{n} i \cdot i! = (n+1)! - 1$

Example 4 $a_1 + (a_1 + d) + (a_1 + 2d) + \cdots + (a_1 + (n-1)d) = \dfrac{n}{2}(2a_1 + (n-1)d)$

Example 5 $1^2 - 2^2 + 3^2 - 4^2 + \cdots + (-1)^{n+1} n^2 = (-1)^{n+1} \dfrac{n(n+1)}{2}$